Question 1 (5 pts). Find the solution of the linear equation

$$y'' - 22y' + 121y = 0$$

that also satisfies $y(0) = 0$ and $y'(0) = 1$.

Solution: We consider the polynomial

$$\lambda^2 - 22\lambda + 121 = (\lambda - 11)^2 = 0.$$ 

This polynomial has $\lambda = 11$ as a root of multiplicity 2. Hence the general solution of the equation is

$$y(t) = \alpha_1 e^{11t} + \alpha_2 t e^{11t}.$$ 

Now we impose

$$y(0) = \alpha_1 = 0.$$ 

Hence $y(t) = \alpha_2 t e^{11t}$. Hence

$$y'(t) = \alpha_2 e^{11t} + 11\alpha_2 t e^{11t}$$ 

so

$$y'(0) = \alpha_2 = 1.$$ 

Question 2 (5 pts). Find the general solution of the linear equation

$$y'' + 12y' + 40y = 0.$$ 

Solution: We consider the polynomial

$$\lambda^2 + 12\lambda + 40 = (\lambda + 6)^2 + 4 = 0.$$ 

The roots are $\lambda = -6 \pm 2i$, hence the general solution is

$$y(t) = \alpha_1 e^{-6t} \cos(2t) + \alpha_2 e^{-6t} \sin(2t).$$