Question 1 (5 pts). Determine if the functions $e^t$ and $e^{2t}$ are linearly independent in $(-\infty, +\infty)$.

Solution: We compute the Wronskian

$$W(e^t, e^{2t})(t) = \det \begin{pmatrix} e^t & e^{2t} \\ e^t & 2e^{2t} \end{pmatrix} = 2e^{3t} - e^{3t} = e^{3t}.$$ 

The functions are linearly independent over the whole $(-\infty, +\infty)$.

Question 2 (5 pts). Determine the value or values of $\lambda$ for which the vectors $(2, -3)$ and $(-3, 5 + \lambda)$ are linearly dependent.

Solution: To determine the value of $\lambda$ for which these vectors are linearly independent we compute the determinant

$$\det \begin{pmatrix} 2 & -3 \\ -3 & 5 + \lambda \end{pmatrix} = 2(5 + \lambda) - 9 = 2\lambda + 1.$$ 

The vectors are linearly dependent when $\det \begin{pmatrix} 2 & -3 \\ -3 & 5 + \lambda \end{pmatrix} = 0$, so we set

$$2\lambda + 1 = 0 \text{ which says } \lambda = \frac{-1}{2}.$$ 

For $\lambda = \frac{-1}{2}$ the vectors above are dependent.