Question 1. 5 pts. Find the Jacobian of the transformation defined through the equations
\[ x(u, v) = u^2 - v^2 \text{ and } y(u, v) = u^2 + v^2. \]

Solution: We first compute
\[ \frac{\partial x}{\partial u} = 2u, \quad \frac{\partial x}{\partial v} = -2v, \quad \frac{\partial y}{\partial u} = 2u, \quad \text{and} \quad \frac{\partial y}{\partial v} = 2v. \]

From here we have
\[ J(u, v) = \left| \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \frac{\partial y}{\partial u} \right| = 8 |uv|. \]

Question 2. 5 pts. Find the length of the curve
\[ \gamma(t) = \left( t, \frac{2}{3} t^{\frac{3}{2}} \right) \]
for \( t \in [0, 3] \).

Solution: Let us find the tangent to the curve first:
\[ \gamma'(t) = (1, t^{\frac{1}{2}}). \]

Then, we find the length of this tangent vector:
\[ |\gamma'(t)| = \left| (1, t^{\frac{1}{2}}) \right| = \sqrt{1 + t}. \]

To find the length of the curve \( \gamma(t) \), where \( t \in [0, 3] \), we integrate
\[ \int_0^3 \sqrt{1 + t} \, dt = \left. \frac{2}{3} (1 + t)^{\frac{3}{2}} \right|_0^3 = \frac{14}{3}. \]